

2.1) Para probar que son TL, debo verificar que cumplan lo siguiente:

$$\textcircled{1} F(u+v) = F(u) + F(v)$$

$$\textcircled{2} F(\lambda v) = \lambda F(v)$$

$$a) T([xyz]^T) = 2z - 3y.$$

$$\textcircled{1} \text{ Tomo } u = [x_1 \ y_1 \ z_1]^T \text{ y } v = [x_2 \ y_2 \ z_2]^T$$

$$T(u) = 2z_1 - 3y_1$$

$$T(v) = 2z_2 - 3y_2$$

$$u+v = [x_1+x_2 \ y_1+y_2 \ z_1+z_2]^T$$

$$T(u+v) = 2(z_1+z_2) - 3(y_1+y_2) \rightarrow$$

$$\rightarrow T(u+v) = (2z_1 - 3y_1) + (2z_2 - 3y_2) = T(u) + T(v) \quad \checkmark$$

$$\textcircled{2} \text{ Tomo } u = [x_1 \ y_1 \ z_1]^T, \lambda \in K$$

$$\rightarrow \lambda u = [\lambda x_1 \ \lambda y_1 \ \lambda z_1]$$

$$T(u) = 2z_1 - 3y_1$$

$$T(\lambda u) = 2 \cdot \lambda z_1 - 3 \cdot \lambda y_1 = \lambda \cdot (2z_1 - 3y_1) = \lambda \cdot T(u) \quad \checkmark$$

Por lo tanto T es TL.

$$b) T([x \ y \ z]^T) = [2z - 3y \ -z + 3x]^T$$

$$\textcircled{1} \text{ Τοποθε } u = [x_1 \ y_1 \ z_1]^T \text{ τότε } T(u) = [2z_1 - 3y_1 \ -z_1 + 3x_1]^T$$

$$\text{ ή τοποθε } v = [x_2 \ y_2 \ z_2]^T \text{ τότε } T(v) = [2z_2 - 3y_2 \ -z_2 + 3x_2]^T$$

$$u+v = [x_1+x_2 \ y_1+y_2 \ z_1+z_2]^T$$

$$T(u+v) = [2(z_1+z_2) - 3(y_1+y_2) \ -(z_1+z_2) + 3(x_1+x_2)]^T \rightarrow$$

$$\rightarrow T(u+v) = [2z_1 - 3y_1 \ -z_1 + 3x_1]^T + [2z_2 - 3y_2 \ -z_2 + 3x_2]^T = T(u) + T(v) \checkmark$$

$$\textcircled{2} \text{ Τοποθε } u = [x_1 \ y_1 \ z_1]^T \text{ τότε } T(u) = [2z_1 - 3y_1 \ -z_1 + 3x_1]^T,$$

$\lambda \in K$

$$\lambda u = [\lambda x_1 \ \lambda y_1 \ \lambda z_1]^T$$

$$T(\lambda u) = [2(\lambda z_1) - 3(\lambda y_1) \ -(\lambda z_1) + 3(\lambda x_1)]^T = \lambda \cdot [2z_1 - 3y_1 \ -z_1 + 3x_1]^T = \lambda \cdot T(u) \checkmark$$

Ποι τα στοιχεία T_2 ει T_L .

$$c) T([x \ y \ z]^T) = [2z - 3y \ -z + 3x \ y - 2x]^T$$

$$\textcircled{1} \text{ Τοποθε } u = [x_1 \ y_1 \ z_1]^T \text{ τότε } T(u) = [2z_1 - 3y_1 \ -z_1 + 3x_1 \ y_1 - 2x_1]^T$$

$$\text{ ή τοποθε } v = [x_2 \ y_2 \ z_2]^T \text{ τότε } T(v) = [2z_2 - 3y_2 \ -z_2 + 3x_2 \ y_2 - 2x_2]^T$$

$$u+v = [x_1+x_2 \ y_1+y_2 \ z_1+z_2]^T$$

$$T(u+v) = [2(z_1+z_2) - 3(y_1+y_2) \ -(z_1+z_2) + 3(x_1+x_2) \ (y_1+y_2) - 2(x_1+x_2)]^T \rightarrow$$

$$\rightarrow T(u+v) = [2z_1 - 3y_1 \ -z_1 + 3x_1 \ y_1 - 2x_1]^T + [2z_2 - 3y_2 \ -z_2 + 3x_2 \ y_2 - 2x_2]^T = T(u) + T(v) \checkmark$$

$$\textcircled{2} \text{ Τοποθε } u = [x_1 \ y_1 \ z_1]^T \text{ τότε } T(u) = [2z_1 - 3y_1 \ -z_1 + 3x_1 \ y_1 - 2x_1]^T,$$

$\lambda \in K$

$$\lambda u = [\lambda x_1 \ \lambda y_1 \ \lambda z_1]^T$$

$$T(\lambda u) = [2(\lambda z_1) - 3(\lambda y_1) \ -(\lambda z_1) + 3(\lambda x_1) \ (\lambda y_1) - 2(\lambda x_1)]^T = \lambda \cdot [2z_1 - 3y_1 \ -z_1 + 3x_1 \ y_1 - 2x_1]^T = \lambda T(u) \checkmark$$

Ποι τα στοιχεία T_3 ει T_L .